Satisfiability of elastic demand in the smart grid
as understood by P. Maillé

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Motivation

Context: consider a given area (say, small city) **producing** and **consuming** energy

Production

- wind turbines
- solar pannels

Consumption

- users with different appliances

- Energy can be bought to cover the extra demand, **but** changes in the quantity bought are limited (ramp-up, ramp-down)
- Production as well as consumption are not perfectly predictable
- Some demand can be delayed

Question: is it possible to delay demand to buy less energy, but avoiding instability?

The mathematical model

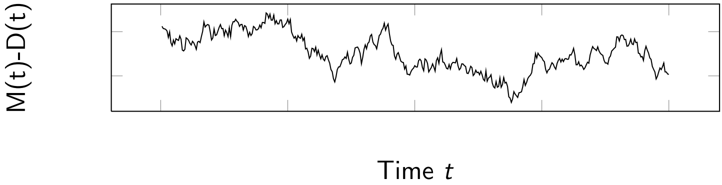
- Time is discretized, indexed by t
- Actual "new" demand $D^a(t)$ is not perfectly predictable

$$D^a(t) = \underbrace{D^f(t)}_{\text{forecast}} + \underbrace{D(t)}_{\text{random (due to weather...)}}$$

- Actual supply $G^a(t)$ is not perfectly predictable

$$G^a(t) = \underbrace{G^f(t)}_{\text{forecast}=D^f(t)+r_0} + \underbrace{M(t)}_{\text{random (due to weather)}} + \underbrace{G(t-1)}_{\text{bought from outside}}$$

- $M(t) - D(t)$ is modeled as an ARIMA(0,1,0) *(discrete version of the Brownian motion)*



What happens if demand exceeds supply?

- The *frustrated* demand is backlogged and will be expressed later, **(being multiplied by** $1 - \underbrace{\mu}_{\text{“evaporation”}<1}$ **at each time slot)**
  - *Positive evaporation*: delayed demand decreases over time (ex: heating system)
  - *Negative evaporation*: delayed demand increases over time (ex: heat pump)

The detailed process:

$$\begin{aligned} \underbrace{E^a(t)}_{\text{Total expressed demand}} &= \underbrace{D^a(t)}_{\text{new demand}} + \underbrace{\lambda}_{1/\lambda=\text{delay}} \underbrace{Z(t)}_{\text{latent backlogged demand}} \\ Z(t+1) &= (1-\mu)Z(t) + \underbrace{[E^a(t) - G^a(t)]^+}_{\text{frustrated demand}} - \lambda Z(t) \end{aligned}$$

Control policy

The control variable: what to buy from outside $G(t)$

BUT limit

$$\underbrace{-\xi \le}_{\text{ramp-up constraint}} H(t) = G(t) - G(t-1) \underbrace{\le \zeta}_{\text{ramp-down constraint}}$$

for some $\xi > 0$ and $\zeta > 0$.

The policy considered: threshold-based control

$$H(t) = \max(\min(\zeta, \underbrace{r^*}_{\text{"margin" objective}} - (\underbrace{G^a(t)}_{\text{supply}} - \underbrace{E^a(t)}_{\text{expressed demand}}), -\xi)$$

Question tackled in the paper: does such a policy work?

Results

Once the control policy is fixed, the two-dimensional process describing $(\underbrace{G^a(t)}_{\text{supply}} - \underbrace{E^a(t)}_{\text{expressed demand}}, \underbrace{Z(t)}_{\text{backlogged demand}})$ is a Markov chain

(discrete-time, general state space)

Theorem

If $\mu > 0$ (positive evaporation), the Markov chain is ergodic. For any initial distribution, the chain converges to its unique invariant probability measure.

In words: the system is stable! (Backlogged demand does not explode if multiplied by $1 - \mu < 1$ at each time slot)

Theorem

If $\mu < 0$ (negative evaporation), the Markov chain is non-positive.

In words: the system is unstable (Backlogged demand explodes if multiplied by $1 - \mu > 1$ at each time slot)

See the 10-page long proof...

Conclusions

- An interesting and rigorous model with
  - Energy production, consumption, and purchase
  - Random parts in demand and supply
  - Two types of impact of delayed demand (reduction or increase)
  - Control variable: energy purchased
- Some analytical results (stability, instability), but no so surprising

Perspectives/extensions:

- Investigating other policies than the threshold-based one
- Applicability in other contexts (demand for content, instead of energy?) $\Rightarrow$ cf Samantha's thesis with *delay-tolerant users*
- Complement the analysis by modeling the costs (of delay) and gains (in energy purchased) of such methods

Potential PhD candidates on those topics?